Vol.7 Issue No. 6 December 2011
Print ISSN 2244-4432 • Online ISSN 2984-7125
National Peer Reviewed Journal
DOI: https://doi.org/10.59120/drj.v7i6.641

Derivation of a Solution to a Linear Diophantine Equation in Three and Four Variables

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Abstract

Diophantine equations, introduced by Diophantus, come in many forms. One of these forms is the linear Diophantine equation discovered by the followers of Diophantus. The main concern of this study is to explore linear Diophantine equations with more than two variables. It also aims to derive the general solutions of linear Diophantine equations in three and four variables using the concept of the Euclidean algorithm and the extended Euclidean algorithm. The results of this study showed that, in deriving the general solutions of three and four variables, the condition for solvability must be satisfied to determine that the equation admits a solution. Then, using the concept of the extended Euclidean algorithm with two variables, the general solutions of linear Diophantine equations with three and four variables can be established. The researcher recommends in working equation with five or more variables using the concept of the extended Euclidean algorithm. In addition, to generate a computer program to find and evaluate the general solutions of linear Diophantine equations.

Keywords: Diophantine equation, Diophantus, Euclidean algorithm, Linear, Variables

Introduction

Diophantine equation was named after the Hellenistic Mathematician Diophantus. He was the first mathematician to introduce symbolism in algebra and sought a method for finding integer solutions to linear indeterminate equations that lack sufficient information to yield a single discrete set of answers (Wolfgang, 2000). A linear Diophantine equation is an equation between two sums of monomials of degree zero or one and takes the form ax + by = = c. If c is the greatest common divisor of a and b, then this is Bezout's identity, and the equation has an infinite number of solutions. These can be found by applying the extended Euclidean algorithm. Then, it follows that there are also infinitely many solutions if c is a multiple of the greatest common divisor of a, and b. However, if c is not a multiple of the greatest common divisor of a and b, then the Diophantine equation ax + by = c has no solutions (Dutta, 2002). The original form of the Diophantine equation ax + by already had a

general solution, and there was a study that established the particular solutions with three variables. Since the solutions of two variables were already organized and the three-variable solutions were only particular solutions, the researcher is now going to present this to use as a basis in establishing the general solutions of three variables in the form of ax + by + cz = d and four variables in the form ax + by + cz + dw = e, respectively. This research intended to derive the solution of the linear Diophantine equation in three and four variables. Particularly, it aimed to answer the following questions: 1. What is the general solution of ax + by =c?, 2. What is the general solution of ax + by + cz =d? 3. What are the conditions to establish the general solutions of four variables in the form ax + by + cz + dw = e as a conjecture?. The main objective of this study was to derive a solution of the linear Diophantine equation in three and four unknown variables. Specifically, this study aimed 1. To present the general solution in the form ax + by = c. 2. To present the general solution in the form ax + by + cz = d. 3. To provide the conditions for establishing the general solutions of four variables in the form ax + by + cz + dw = e as a conjecture. Linear Diophantine equations are another important topic in number theory. Since the original equation deals only with two variables, the researcher made use of this idea to extend it to three and four variables and to establish its general solutions. The result of this study will give additional knowledge to the mathematics students and mathematics enthusiasts here in DOSCST who are concerned with the common type of problem which involved more variables. Moreover, this study focused on presenting the general solutions of two variables and deriving the general solutions of three variables and four variables. The general solutions of four variables were established as a conjecture based on the conditions. It utilized the concepts of two unknowns and the extended Euclidean algorithm.

For the purpose of this study, the following terms were defined. Arbitrary integers refer to integers that can assume positive or negative values. Divisibility: An integer b is said to be divisible by an integer a, where $a \neq 0$, if there exists some integer c such that b = ac. Euclid's algorithm is an algorithm that determines the greatest common divisor of any two integers. Extended Euclidean Algorithm finds integers a and b such that gcd (a,b) = am + bn such m and n element of Z.

Methodology

To present the General Solution of ax + by = c.

In presenting the general solution of a linear Diophantine equation in two variables in the form of ax + by = c, it must satisfy the condition for solvability to know whether the solutions exist, that is, gcd (a, b) divides c. Then, utilize the Euclidean algorithm (see preliminary concept) to get the particular solution. Finally, if $x_\circ$ and $y_\circ$ is any particular solution, then the general solution of x and y would be determined using the concept of divisibility.

To present the General Solution of ax + by + cz = d.

In finding the general solutions of three variables in the form ax + by + cz = d, the condition for solvability should be first satisfied, where gcd (a, b, c) divides d. Next, we find gcd (a, b) and factor it out from the original equation. Then, reduce these three variables into two variables by letting the expressions of x and y be new variables, say w. After that, applying the concept of the extended Euclidean algorithm with two variables, we can generate its solutions.

To Present the General Solution of ax + by + cz + dw = e

On the other hand, determining the condition of the general solutions in four variables in the form ax + by + cz + dw = e contained the same process in three variables as stated above.

Print ISSN 2244-4432 • Online ISSN 2984-7125 DOI: https://doi.org/10.59120/drj.v7i6.641

Results and Discussion

Derivation of General Solutions in Linear Diophantine Equations with Two Variables

The linear Diophantine equation of two variables

$$ax + by = c$$

Assume that a and b are both nonzero

Let

$d =$ gdc (a,b)

→ d|a and d|b

→ a = dr and b = ds for some integers r and s

Suppose that we have a solution (x_0, y_0) to the equation

→ $ax_0 + by_0 = c = drx_0 + d$

→ $d|\ (ax_0 + by_0)$

→ d|c

Conversely, suppose d|c, say c = dt, then there exist integers x_0 and y_0 such that

$$d = ax_0 + by_0$$

Hence, multiplying by t, we obtain

→ $(ax_0 + by_0 = d)\ t$

→ $ax_0\ t + by_0\ t = dt$

→ $x = (tx_0)$ and $y = (ty_0)$ is a particular solution.

The linear Diophantine equation ax + by = c has a solution if and only if d|c, where d = gcd (a,b). If x_0, y_0 is any particular solution of this equation, then all other solutions is given by

$$x = x_0 + \frac{b}{d}t \qquad \text{and} \qquad y = y_0 - \frac{a}{d}t$$

Proof:

Suppose that a solution x_0 y_0 of ax + by = c is known. If x' and y' are any other solutions, then

→ $ax_0 + by_0 = c = ax' + by'$

→ $ax' - ax_0 = by_0 - by'$

Then there exist r and s ϵ Z such that r and s are relatively prime, since a = dr and b = ds.

→ $rd\ \frac{(x' - x_0) = sd\ (y' - y_0)}{d}$

→ $r(x' - x_0) = s(y' - y_0)$

Now, $r = \frac{a}{b}$ and $s = \frac{b}{d}$ Then, $gcd\ (r,s) = 1$

→ $r|s\ (y_0 - y'), s|r\ (x' - x_0)$

By Euclid's lemma, r| (y_0 - y'), s| (x' - x_0), now, consider r| (y_0 - y'),

→ (y_0 - y') = rt for some t ϵ

→ y_0 - y' = rt

→ y' = y_0 - rt, where y' = y

Thus,

$y = y_0 = rt$ since $r = \frac{a}{d}$

$y = y_0 - \frac{a}{d}t$

Next consider s| (x' - x_0) = st

→ (x' - x_0) = st

→ x' - x_0 = st

→ x' = x_0 = st, where x' = x

we have,

$x = x_0 + st$ since $s = \frac{b}{d}$

$x = x_0 + \frac{b}{d}t$

Illustrative example:

1. Find all solutions of 77x + 42y = 35

First, we calculate gcd (77, 42) using the Euclidean algorithm.

77 = 42*1 + 35

42 = 35*1 + 7

35 = 7*5 + 0

So

gcd (77, 42) = 7

Since 7 divides 35, this means that the linear Diophantine equation does have integer solutions. To actually find the solutions we first reverse the steps in the Euclidean algorithm.

7 = 42 – 35

= 42 – (77 -42)

[= (-1) *77 = 2*42]5

So we take u = -1 and v = 2 One solution is then,

x_0 = (-1) *35/7 = -5 y_0 = 2*35/7 = 10

All the solutions are given by

x = x_0 + (42/7) t = -5 + 6t y = y_0 - (77/7) t 10 – 11t

where t € Z.

Derivation of General Solutions in Linear Diophantine Equations with Three Variables

Based on the researcher's observations, consider the following in deriving the general solutions in three variables.

By Theorem 2.4, a linear Diophantine equation in the form of ax + by + cz = d has a solution exist, if and only if e|d, where e = gcd(a,b,c).

The linear Diophantine equation with three variables,

ax + by + cz = d

Factor out the equation

Print ISSN 2244-4432 • Online ISSN 2984-7125 DOI: https://doi.org/10.59120/drj.v7i6.641

$$gdc\ (a,b)(\frac{ax}{gcd\ (a,b)} + \frac{by}{gcd\ (a,b)}) + cz = d$$

Let

$$w\ = (\frac{ax}{gcd\ (a,b)} + \frac{by}{gcd\ (a,b)})$$

o

$$w\ = (\frac{md}{gcd\ (e,c)} + \frac{ck}{gcd\ (e,c)})$$

$$z\ = (\frac{nd}{gcd\ (e,c)} + \frac{ek}{gcd\ (e,c)})$$

Next, find x and y
The equation becomes

$$\frac{ax}{e,} + \frac{by}{e} = w$$

Using the extended Euclidean algorithm

$$gdc\ (\frac{a}{e} + \frac{b}{e}) = g = \frac{a}{e}m + \frac{b}{e}n_1$$ for some integers m_1 and $n_1 \epsilon Z$.

The condition for solvability must be satisfied; that is, gdc $(\frac{a}{e}, \frac{b}{e}) | w$

Again, utilizing the concept in Appendix A, the general solutions of x and y

$$\rightarrow x = m^1\left(\frac{w}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}\right) + \frac{b/e}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}1$$

$$\rightarrow y = n^1\left(\frac{w}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}\right) + \frac{a/e}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}1$$

Therefore, all solutions are given by:

$$x = m_1\left(\frac{w}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}\right) + \frac{bt/e}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}$$ for some $m_1 \epsilon Z$

$$y = n_1\left(\frac{w}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}\right) + \frac{at/e}{gcd\left(\frac{a}{e},\frac{b}{e}\right)}$$ for some $n_1 \epsilon Z$

$$z\ = (\frac{nd}{gcd\ (e,c)} - \frac{ek}{gcd\ (e,c)})$$ for some n ϵZ

where k and / are arbitrary integers, e = gcd(a,b) and w\ = $\frac{md}{gcd\ (e,c)} + \frac{ck}{gcd\ (e,c)}$

Illustration:

To find solutions to a linear Diophantine Equation in three variables, consider 15x + 12y + 30z = 24.

Test the condition for solvability such that e divides d, where e = gcd(a,b,c).
So

gcd(15,12,30) = 3

Since 3 divides 24, this means that the linear Diophantine equation in three variables does have integer solution.

To find the solutions.

We have y

gcd(15,12) (5x+4y) + 30z = 24

Let w = 5x + 4y

The equation becomes

3w + 30z = 24

Applying the Extended Euclidean Algorithm

gcd (3,30) = 3(-9) + 30(1)

Based in the appendix A the general solutions of r and y x =

$$w = \frac{-9(24)}{3} + \frac{30}{3}k$$

$$w = -72 + 10k$$

$$z = \frac{1(24)}{3} + \frac{3k}{3}$$

$$z = 8 - k$$

Next, find x and y

5x + 4y + 72 + 10k

Applying the Extended Euclidean algorithm

gdc (5,4) = 1 = 5(1) + 4(-1)

Some gdc (5,4) | - 72 + 10k, the equation is solvable

Again, based on Appendix A, the general solutions of x and y

$$x = 1(\frac{-72 + 10k}{1}) + \frac{4}{1}I$$

$$x = -72 + 10k + 4I$$

$$y = -1(\frac{-72 + 10k}{1}) + \frac{5}{1}I$$

$$y = 72 + 10k - 5I$$

All general solutions are

x = - 72 + 10k + 4l

y = 72 + 10k – 5I

Print ISSN 2244-4432 • Online ISSN 2984-7125 DOI: https://doi.org/10.59120/drj.v7i6.641

$$z = 8 - k$$

where k and I are arbitrary integers.

Substitute the value of k and/ in the original equation.

$$15x + 12y + 30z = 24$$

where k and I are equal to 0,1,2

If k = 0 and l = 0

$$15(-72) + 12(72) + 30(8) = 24$$
$$-1080 + 864 + 240 = 24$$
$$24 = 24$$

If k = 1 and I = 1

$$15(-58) + 12(57) + 30(7) = 24$$
$$-870 + 684 + 210 = 24$$
$$24 = 24$$

Derivation of General Solutions in Linear Diophantine Equation with Four Variables

Based on the researcher's exploration in getting the general solutions of four variables, the researcher was able to come up with the following:

By Theorem 2.4, the linear Diophantine equation in four variables $ax + by + cz + dw = e$ has a solution if and only if e|f, where $f = gcd(a,b,c,d)$.

The linear Diophantine equation in four variables,

$$ax + by + cz + dw = e$$

Factor out the expression

$$gcd(a,b,c)\left(=\frac{ax}{gdc(a,b,c)}+\frac{by}{gdc(a,b,c)}+\frac{cz}{gdc(a,b,c)}\right)+dw=e$$

Let

$$v\left(=\frac{ax}{gdc(a,b,c)}+\frac{by}{gdc(a,b,c)}+\frac{cz}{gdc(a,b,c)}\right)$$

The equation becomes

$$gcd(a,b,c)\,v + dw = e$$

Let

$$f = gcd(a,b,c)$$
$$\rightarrow fv + dw = e$$

Applying the extended Euclidean Algorithm

$$gcd(f, d) = g = fm + dn \quad \text{for some m and n } \epsilon Z$$

Based on Appendix A, we will use the method for finding the general solutions of x and y to find the general solutions of v and w.

Then, the general solutions of v and w

$$v = \frac{md}{gcd(f,d)} + \frac{dk}{gcd(f,d)}$$

$$w = \frac{nd}{gcd(f,d)} + \frac{dk}{gcd(f,d)}$$

Next, find x, y, and z

We have $\frac{ax}{f} + \frac{by}{f} + \frac{cz}{f} = v$

Factor out the equation

$$gcd(\tfrac{a}{f}, \tfrac{b}{f})\,(\tfrac{ax|f}{gcd(\frac{a}{f},\frac{b}{f})} + \tfrac{by|f}{gcd(\frac{a}{f},\frac{b}{f})}) + \tfrac{cz}{f} = v$$

Let $\mathrm{j} = (\frac{\mathrm{ax|f}}{\gcd(\frac{\mathrm{a}}{\mathrm{f}},\frac{\mathrm{b}}{\mathrm{f}})} + \frac{\mathrm{by|f}}{\gcd(\frac{\mathrm{a}}{\mathrm{f}},\frac{\mathrm{b}}{\mathrm{f}})})$

and $\rightarrow \gcd(\frac{\mathrm{a}}{\mathrm{f}},\frac{\mathrm{b}}{\mathrm{f}}) = \mathrm{i} = \frac{\mathrm{a}}{\mathrm{f}}m_2 + \frac{\mathrm{b}}{\mathrm{f}}n_2$ for some m_2 and n_2

The condition for solvability must be satisfy, that is$\gcd(\frac{\mathrm{a}}{\mathrm{f}},\frac{\mathrm{b}}{\mathrm{f}})\mathrm{i}|$.

Repeat the concept of getting the general solutions of j and z to find the general solutions of x and y.

$$\rightarrow x = m_2\,(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}) + \frac{bp|f}{gcd(\frac{a}{f},\frac{b}{f})}$$

$$\rightarrow y = n_2\,(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}) - \frac{ap|f}{gcd(\frac{a}{f},\frac{b}{f})}$$

Therefore, these are the conjectures of the general solutions in four variables.

$$x = m_2\,(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}) + \frac{bp|f}{gcd(\frac{a}{f},\frac{b}{f})} \qquad \text{for some } m_2 \in Z$$

$$y = n_2\,(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}) - \frac{ap|f}{gcd(\frac{a}{f},\frac{b}{f})} \qquad \text{for some } n_2 \in Z$$

$$z = n_1\,(\frac{v}{gcd(q,\frac{c}{f})}) - \frac{q|}{gcd(q,\frac{c}{f})} \qquad \text{for some } n_1 \in Z$$

$$w = \frac{md}{gcd(f,d)} - \frac{q|}{gcd((f,d)} \qquad \text{for some n} \in Z$$

where k,l, and p are arbitrary integers, f = gcd(a,b,c),

$$\mathrm{j} = \frac{v}{gcd(q,\frac{c}{f})} - \frac{d|f}{gcd(q,\frac{c}{f})} \text{ and } \mathrm{v} = \frac{md}{gcd(f,d)} - \frac{dk}{gcd((f,d)}$$

Illustration

In finding the solutions of a linear Diophantine equation in four variables, consider the equation *172x + 20y + 22z + 11w = 1000.*

Test the condition for solvability where f divides e such that *f = gcd (a,b,c,d)*

So *gcd (172, 20, 22, 11) = 1*

Since 1 divide 1000, this means that the linear Diophantine equation in four variables does have integer solutions but *gcd (172, 20, 22) = 2* Now

gcd (172, 20, 22)(86x + 10y + 11z) + 11w = 1000

Print ISSN 2244-4432 • Online ISSN 2984-7125 DOI: https://doi.org/10.59120/drj.v7i6.641

Let

$v = 86x + 10y + 11z$

The equation becomes

$2v + 11w = 1000$

Applying the extended Euclidean Algorithm

$gcd\ (2, 11) = 1 = 2(-5) + 11(1)$

Based on Appendix A, the concept of getting the general solutions of x and y is used to find the general solutions of v and w.

$$v = \frac{-5\,(1000)}{1} + \frac{11}{1}\,k$$

$$v = -5000 + 11k$$

$$v = \frac{1(1000)}{1} - \frac{2k}{1}$$

$$w = 1000 - 2k$$

Find x, y and z

We have $86x + 10y + 11z = -5000 + 11k$

Factor out the expression

$gcd\ (86, 10)(43x + 5y) + 11z = -5000 + 11k$

Let

$j = 43x + 5y$

The equation becomes

$2j + 11z + -5000 + 11k$

Applying the extended Euclidean algorithm

$gcd\ (2, 11) = 1 = 2(-5) + 11(1)$

Since gcd (2,11) | -5000 + 11k, the solution is solvable.

Repeat the concept in Appendix A the general solutions of j and z

$$j = -5(\frac{-5000 + 11k}{1}) + \frac{11}{1}$$

$$j = 25000 - 55\mathrm{k} + 11I$$

$$z = 1(\frac{-5000 + 11k}{1}) + \frac{2}{1}I$$

$$\mathrm{z} = -5000 + 11\mathrm{k} - 2I$$

Then, x and y

The equation becomes $43x + 5y + 25000 - 55k + 11I$

Applying the extended Euclidean algorithm

$gcd\ (43,5) = 1 = 43(2) + 5(-17)$

Since gcd (43,5) 2500 – 5k + 11k, the solution is solvable

Repeat the concept in Appendix A, the general solutions of x and y.

$$x = -5\left(\frac{-25000 - 55k + 11I}{1}\right) + \frac{5}{1}m$$

$$x = 50000 - 110k + 22I + 5m$$

$$y = -17\left(\frac{-25000 - 55k + 11I}{1}\right) + \frac{43}{1}m$$

$$y = 425000 - 935k - 187I - 43m$$

All general solutions are:

$$x = 50000 - 110k + 22I + 5$$

$$y = -425000 + 935k - 187I - 43m$$

$$z = -5000 + 11k - 2I$$

$$w = 1000 - 2k$$

where *k,* I and p in the original equation.
Substitute the value of k, I and p in the original equation.

$$172x + 20y + 22x + 11w = 1000$$

Where *k,* I and p are equal to 0,1,2.
If $k = 0,1 = 0$ and $p = 0$

$$172(50000) + 20(-425000) + 22(-5000) + 11(1000) = 1000$$
$$8600000 - 8500000 - 110000 + 11000 = 1000$$
$$1000 = 1000$$

If $k = 1$, $I = 1$ and $p = 1$

$$172\,(49917) + 20(-424295) + 22(-4991) + 11(998) = 1000$$
$$8585724 - 8485900 - 109802 + 10978 = 1000$$
$$1000 = 1000$$

If $k = 0,1$, $I = 1$ and $p = 2$

$$172(50032) + 20(-425273) + 22(-5002) + 11(1000) = 1000$$
$$8605504 - 8505460 - 110044 + 11000 = 1000$$
$$1000 = 1000$$

Conclusion

After the derivation of solutions was done, the following conclusions were drawn:

1. The general solutions of linear Diophantine equation in the form $ax + by = c.\backslash$

$$\text{has } x = x_0 + \frac{b}{d}t \text{ and } y_0 + \frac{a}{d}t \text{ for some } t \in Z.$$

Based in the concept of extended Euclidean algorithm, the general solutions of linear Diophantine equation in the form $ax + by + cz = d$ and $ax + by + cz + dw = e$ are the following.

2. For $ax + by + cz = d$.

$$x = m_1\left(\frac{w}{gcd(\frac{a}{e},\frac{b}{e})}\right) + \frac{bl|e}{gcd(\frac{a}{e},\frac{b}{e})} \qquad \text{for some } m_1 \in Z$$

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$$y = n_1\left(\frac{w}{gcd(\frac{a}{e},\frac{b}{e})}\right) - \frac{al|e}{gcd(\frac{a}{e},\frac{b}{e})} \qquad \text{for some } n_1 \in Z$$

$$z = \frac{nd}{gcd(e,c)}) - \frac{ek}{gcd(e,c)} \qquad \text{for some n } \epsilon Z$$

where *k* and *I* are arbitrary integers

3. For $ax + by + cz + dw = e$

$$x = m_2\left(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}\right) + \frac{bp|f}{gcd(\frac{a}{f},\frac{b}{f})} \qquad \text{for some } m_2 \in Z$$

$$y = n_2\left(\frac{j}{gcd(\frac{a}{f},\frac{b}{f})}\right) - \frac{ap|f}{gcd(\frac{a}{f},\frac{b}{f})} \qquad \text{for some } n_2 \in Z$$

$$z = n_1\left(\frac{v}{gcd(q,\frac{c}{f})}\right) - \frac{q|}{gcd(q,\frac{c}{f})} \qquad \text{for some } n_1 \in Z$$

$$w = \frac{md}{gcd(f,d)} - \frac{q|}{gcd((f,d)} \qquad \text{for some n } \epsilon Z$$

where *k*,*1* and *p* are arbitrary integers.

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